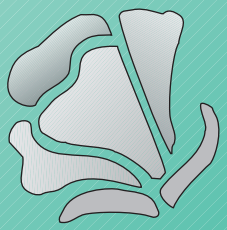
Teacher Guide for Mathematics



Golden State Examination

GSE

First-year Algebra

Geometry

High School Mathematics

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2000

CALIFORNIA DEPARTMENT OF EDUCATION

This document has been prepared by the San Luis Obispo County Office of Education under contract with the California Department of Education. For information about the Golden State Examination testing dates, registration materials, and procedures or information about the Golden State Seal Merit Diploma, contact:

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Introduction

Using the Golden State Examination Teacher Guide

The *Golden State Examination Teacher Guide* has been developed to provide essential information and preparation guidelines for teachers. The guide is intended to serve as an instructional aid in the classroom. It is divided into the following sections:

Test Content — describes the content upon which the Golden State Examination (GSE) questions have been developed.

Test Structure — describes the format of the test.

Scoring Guide — outlines the criteria used to score written-responses problems.

Sample Questions — includes a variety of sample questions, representing the types of questions found on the examinations.

Student Work — provides examples of student responses to written-response problems with teacher commentary.

Teachers are encouraged to reproduce portions or all of the guide for classroom use. Districts/schools also can use these materials for staff development.

Student Eligibility

The GSE in high school mathematics is given during the winter test administration. The first-year algebra and geometry exams are given in the spring. Each examination may be taken only once.

Students who are enrolled in first-year algebra or geometry at the time the examinations are given and students who have taken the courses since the spring 1999 test administration may take the exams. Students completing a second-year integrated mathematics course may take either the first-year algebra exam, the geometry exam, or both. The high school mathematics examination is intended for students who have completed three years of high school mathematics. The exam covers the content included in the standards adopted by the State Board of Education for algebra I, geometry, algebra II, and probability and statistics. It is recommended that students taking the high school mathematics examination thoroughly review the test content for first-year algebra, geometry, and high school mathematics.

Test Preparation

Students should have a firm foundation in the knowledge and skills needed to master the subject area. Sound preparation for the Golden State Examinations should include classroom assignments that allow students to use and test their knowledge.

Students preparing for the examinations need to be able to articulate the major ideas and concepts in the subject area being tested. They must be able to analyze information, apply knowledge, solve problems, and explain their solutions.

In accordance with the 1999–2000 state budget, the content of the current Golden State Examinations will be reviewed to ensure their full alignment to the content standards adopted by the State Board of Education. Teachers should review their curriculum and instructional activities for alignment to these standards.

Reporting Results

All students who complete both sessions of the GSE in algebra, geometry, or high school mathematics receive an individual report of results. Scores for the multiple-choice and written-response portions of the exam are combined to produce the student's overall achievement level. There are six achievement levels. Students who achieve level six are awarded high honors; those who achieve level five are awarded honors; and those who achieve level four are awarded recognition. Students who achieve levels three or below are acknowledged for their participation. Results for the winter administration are mailed to districts in May; those for the spring administration are mailed in October.

Resource Document

The Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve are available at http://www. cde.ca.gov/board/board.html on the Internet. Copies also are available from the Publications Division, Sales Office, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099 ext. 6.

Other Resources

The *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* (1992) is available from the Publications Division, Sales Office, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099 ext. 6.

The *Standards in Mathematics and English for California High School Graduates* are available from the Intersegmental Coordinating Council, 560 J Street, Suite 390, Sacramento, CA 95814. This document can also be found at http://www.otan.dni.us/certicc on the California Education Round Table Web site.

Testing schedules and other information are available from the GSE coordinator in your district office, county office of education, or the California Department of Education at http://www.cde.ca.gov/cilbranch/sca on the Internet.

Test Content for First-year Algebra, Geometry, and High School Mathematics

The content of the Golden State Examinations in first-year algebra, geometry, and high school mathematics is based on the *Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve* adopted by the State Board of Education. For additional details about the topics covered on the exams, teachers should refer to these adopted standards. A complete listing of the mathematics standards is available at http://www.cde.ca.gov/board/board.html on the Internet.

Other useful source documents include the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* and the *Standards in Mathematics and English for California High School Graduates*, developed by the California Education Round Table.

First-year Algebra

The GSE in first-year algebra measures the use of basic algebraic skills and justifications in solving problems. Topics covered in the examination include but are not limited to:

- Algebraic representation: identify and use the arithmetic properties of integers, rational, irrational, and real numbers for evaluation and simplification of algebraic expressions, equations, and inequalities (including radicals, scientific notation, factoring, exponents, opposites, reciprocals, patterns, monomials, polynomials)
- Equations, inequalities, and systems of equations (including linear and quadratic equations, factoring up to third degree, completing the square, quadratic formula, absolute value, proportion)
- Graphs of linear and quadratic equations and inequalities (parallel and perpendicular relationships, intercepts, intersection, distance formula, midpoint, alternative forms of linear equations and inequalities, predictions)
- Geometric relationships (including similarity, perimeter, area, volume, Pythagorean Theorem)
- Probability and statistics (including independent events, mean, median, mode, scattergrams, histograms, stem and leaf plots, discrete math)
- Applications of the knowledge and skills of algebra, using appropriate problem solving strategies

Geometry

Computation, use of algebra, problem solving, proof (formal and informal), and applications are integrated throughout the GSE in geometry. Topics covered in the examination include but are not limited to:

- Relationships among angles, lines, planes, exterior angles, triangle inequalities related to parallel lines, intersecting lines, and polygons
- Triangle and trigonometric relationships (including Pythagorean relationships, similarity, congruence, special triangle relationships, ratios: sine, cosine, tangent, law of sines, area, and perimeter of triangles)
- Properties of polygons other than triangles (including quadrilaterals; polygons with five or more sides; relationships within polygons; sides, angles, midpoints, diagonals; area, perimeter; similarity)
- Circles (angle and segment relationships, basic relationships involving circles, area, circumference, sectors, arc measure, arc length)
- Coordinate and transformational geometry (including linear, non-linear, translations, reflections, rotations, dilations)
- The properties of three dimensional figures (including angles, surface area, and volume of prisms, pyramids, cylinders, cones, and spheres)
- Geometric probabilities (determine theoretical and experimental probabilities, and make predictions about events using geometric representations)

High School Mathematics

The GSE in high school mathematics covers the major strands of algebra I, geometry, algebra II, and probability and statistics. Topics covered in the examination include but are not limited to:

- Algebra I: the skills and concepts summarized in the content description for the GSE in first-year algebra
- Geometry: the skills and concepts summarized in the content description for the GSE in geometry
- Algebra II: expansion of the mathematical content and concepts of algebra I and geometry, including, but not limited to, working with algebraic solutions of problems in various content areas (e.g., systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system)
- Probability and statistics: the skills and concepts covered in an introduction to the study of probability, the processing of statistical information, sampling and statistical estimation, interpretation of data and graphs, and fundamental statistical problem solving.

Test Structure for First-year Algebra, Geometry, and High School Mathematics

The Golden State Examinations in first-year algebra, geometry, and high school mathematics are two-part examinations, administered in two 45-minute sessions.

Session one consists of multiple-choice questions designed to assess the student's breadth of knowledge. The questions emphasize concepts, principles, analysis, and the application of basic processes. The multiple-choice questions may require students to make connections among mathematical concepts or to organize information to arrive at the correct answer.

The multiple-choice portion of the examinations is machine-scored. Sample multiple-choice questions, similar to those on the examinations, are on pages 8–11 for first-year algebra, pages 16–21 for geometry, and pages 25–27 for high school mathematics. Answer keys are provided.

Session two consists of written-response problems that require students to apply their mathematical skills and knowledge. Students are given problems and asked to provide a correct solution with all steps of their solution clearly shown. Students are also required to provide an explanation of how they arrived at the solution.

The written-response portion of the examinations is scored by experienced mathematics teachers and other professionals in the field. Sample written-response problems with student work and teacher commentary are presented on pages 12–15 for first-year algebra, pages 22–24 for geometry, and pages 28–30 for high school mathematics.

Teachers are encouraged to duplicate this guide for student use and to have students test themselves with the sample questions and problems.

Additional Information

A scientific or graphing calculator may be used. Students are encouraged to bring their own calculators to use during the examination. Minicomputers, pocket organizers, or calculators with QWERTY (typewriter) keyboards are not allowed. Calculators cannot be shared.

Scoring Guide for First-year Algebra, Geometry, and High School Mathematics

The written-response portion of the Golden State Examinations in first-year algebra, geometry, and high school mathematics is scored using criteria based on the general scoring guide below.

A detailed training package for scorers, addressing what students are expected to accomplish, is developed to score each student response.

Score Point 4

The student response thoroughly accomplishes the task. The response:

- shows thorough understanding and use of the central mathematical idea(s)
- includes appropriate and accurate mathematical computations
- presents mathematical knowledge and ideas clearly and skillfully, using combinations of mathematical symbols and/or visual means as supporting evidence

Score Point 3

The student response substantially accomplishes the task. The response:

- shows an essential grasp of the central mathematical idea(s)
- includes appropriate and generally correct mathematical computations
- presents mathematical knowledge and ideas clearly with supporting evidence

Score Point 2

The student response partially accomplishes the task. The response:

- shows a limited grasp of the central mathematical idea(s)
- may include incomplete and/or misdirected mathematical computations
- presents mathematical knowledge and ideas in an unclear manner or without supporting evidence

Score Point 1

The student response makes little or no progress toward accomplishing the task. The response:

- shows little or no grasp of the central mathematical idea(s)
- includes mathematical computations that are incorrect or inappropriate
- presents mathematical knowledge and ideas in a barely (if at all) comprehensible manner

Sample Multiple-choice Questions for First-year Algebra

Directions

Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Incomplete erasures may be read as intended answers.

You should have available two or three No. 2 pencils with erasers and a ruler or a straightedge. You also may have a calculator, including a scientific or graphing calculator. Minicomputers, pocket organizers, or calculators with QWERTY (typewriter) keyboards are not allowed. You are not allowed to share your calculator with other students.

Work as rapidly as you can without sacrificing accuracy. Do not spend too much time on a question that seems too difficult. Answer the easier questions first; then return to the harder ones. Try to answer every question even if you have to guess.

Notes: (1) Figures that accompany problems are drawn as accurately as possible EXCEPT when it is stated that a figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(2) All numbers used are real numbers. All algebraic expressions represent real numbers unless otherwise indicated.

1. If $a = -3$, $b = 2$, and $d = -2$, then find the value of $\dfrac{a^2b - d}{a - d}$.

   A. -20 B. -6 C. -4 D. 4

2. Solve the following equation and give the answer to the nearest hundredth.

   $5x + 2 = 17 - 3(x - 7)$

   A. -0.75 B. -0.25 C. 4.50 D. 5.00

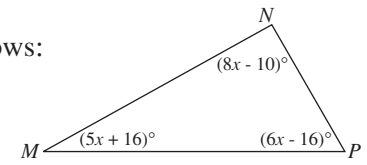
3. The value in cents of $(3n + 7)$ quarters is

   A. $3n + 7$ B. $15n + 35$ C. $30n + 70$ D. $75n + 175$

4. ΔMNP has angles with the indicated measures. The order of the angles from smallest to largest is as follows:

   A. *P, N, M* B. *N, M, P*

   C. *M, P, N* D. *P, M, N*



(Note: Figure not drawn to scale.)

5. Toya has only 5 dimes and 7 nickels in her pocket. She randomly takes out 2 coins. What is the probability that she is holding 15 cents?

A. $\frac{1}{35}$ B. $\frac{35}{144}$ C. $\frac{35}{132}$ D. $\frac{35}{66}$

6. For what value(s) of x is y a positive number in $y = -x^2 - 4x + 5$?

A. $x < -5$ B. $x < 1$ C. $-5 < x < 1$ D. $x < -5$ or > 1

7. Simplify: $\left(\frac{3m^2n}{2p^0q^{-4}}\right)^2$.

A. $\frac{3m^4n^2}{2q^8}$ B. $\frac{9m^4n^2}{4q^8}$ C. $\frac{9m^4n^2q^8}{4}$ D. $\frac{9m^4n^2q^8}{4p^2}$

8. Which of the following equations represents a line passing through $(-2, 3)$ and $(4, -1)$?

A. $2x - 3y = -13$ B. $2x + 3y = 5$ C. $3x - 2y = -12$ D. $3x + 2y = 0$

9. If $(x - 4)$ is a factor of $2x^2 - 5x + p$, what is the value of p?

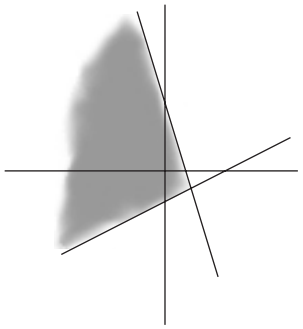
A. -52 B. -12 C. 12 D. 52

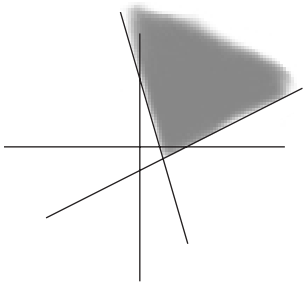
10. Sandy and Chris were the only two candidates for class president. If 379 votes were cast and Chris received 75 more votes than Sandy, how many votes did Chris receive?

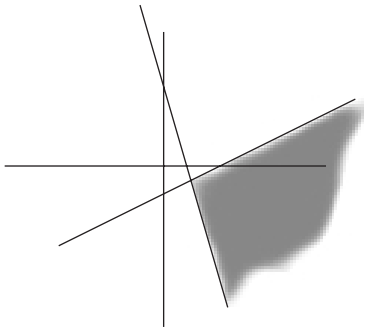
A. 76 B. 152 C. 227 D. 304

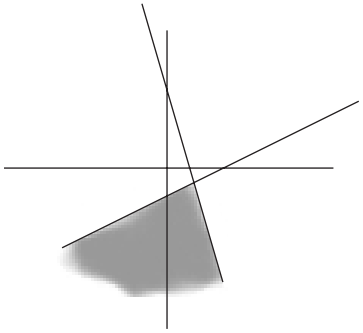
11. Which of the following represents the graph of this system of inequalities?

$3x + y \leq 3$
$x - 2y \geq 2$

A. 

B. 

C. 

D. 

(Note: Figure not drawn to scale.)

12. If $\frac{2x-3}{4} = \frac{x+5}{3}$, then $x =$

A. $-\frac{29}{2}$ B. $-\frac{11}{2}$ C. $\frac{11}{2}$ D. $\frac{29}{2}$

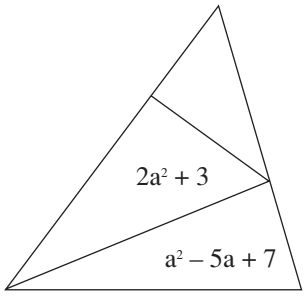
13. What is the perimeter of the triangle with vertices (–15, 0), (6, 0), and (0, 8)?

A. 29 B. 38 C. 48 D. 56

14. Which equation represents a line that is parallel to the line represented by $4x - 7y = 11$ and passing through (3, 2)?

A. $4x - 7y = 2$ B. $4x - 7y = -2$ C. $7x - 4y = 13$ D. $7x - 4y = 29$

15. The total area of the figure at the right is $5a^2 - a + 4$. If two of the regions have the areas shown, what is the area of the remaining region?



(Note: Figure not drawn to scale.)

A. $2a^2 + 4a - 6$ B. $2a^2 - 6a + 14$
C. $2a^2 + 4a + 6$ C. $2a^2 - 4a + 6$

16. Mary has two numbered cubes. The sides of one cube are numbered 1, 2, 3, 4, 5, and 6. The sides of the other cube are numbered 2, 4, 6, 8, 10, and 12. When the cubes are tossed, what is the probability that the sum of the two numbers on top are even?

A. $\frac{3}{12}$ B. $\frac{9}{36}$ C. $\frac{1}{2}$ D. $\frac{9}{12}$

17. A line passes through (4, –1), (–2, –5), and (x, 3). What is value of x?

A. 1 B. 7 C. 10 D. 13

18. For which values of x is the equation $\sqrt{6x-5} = x$ true?

A. 1 only B. 5 only C. 1 and 5 D. No real value

19. If (x, y) is the solution of the system of equations $\begin{cases} 2x - y = 1 \\ 4x - y = 5 \end{cases}$, then $x + y =$

A. 2 B. 3 C. 5 D. 8

20.

x	2	4	8	12
y	– 4	–1	5	11

Based on the values shown in the table above, what is the value of y when $x = 148$?

A. 141 B. 215 C. 222 D. 437

First-year Algebra Answer Key

1. A	4. D	7. C	10. C	13. C	16. C	19. C
2. C	5. D	8. B	11. D	14. B	17. C	20. B
3. D	6. C	9. B	12. D	15. A	18. C	

Sample Written-response Problem for First-year Algebra

Problem

The science class was studying springs. They set up a large spring and put a ball on top. Then they used a device that pushed the spring down, compressing it. When they let go, the ball shot into the air. Each group did several experiments. They measured how far they had lowered the ball by pushing the spring down (the amount of compression), and then they measured how high the ball went. Here is what one group recorded:

Amount of compression	Height the ball reached when shot upward (estimated to the nearest 5 cm)
4 cm	20 cm
5 cm	30 cm
7 cm	55 cm
9 cm	90 cm
10 cm	110 cm

Make a graph of this information.

Develop a rule that uses the amount of compression to figure out how high the ball went. Clearly describe your rule.

Make a prediction for how high the ball will go if the spring is compressed by 15 cm. Explain carefully how you made this prediction.

Remember to show all parts of your solution and explain how you arrived at your answer.

What Students Are Expected to Accomplish Mathematically

Students are asked to construct a graph, analyze a given set of data, and develop a rule for the data. The students must also make a prediction based on the data relating to the height that a ball will travel when released by a compressed spring. The rule and the prediction must be substantiated.

To thoroughly accomplish this task, students must successfully address all three required tasks: the graph, the rule, and the prediction.

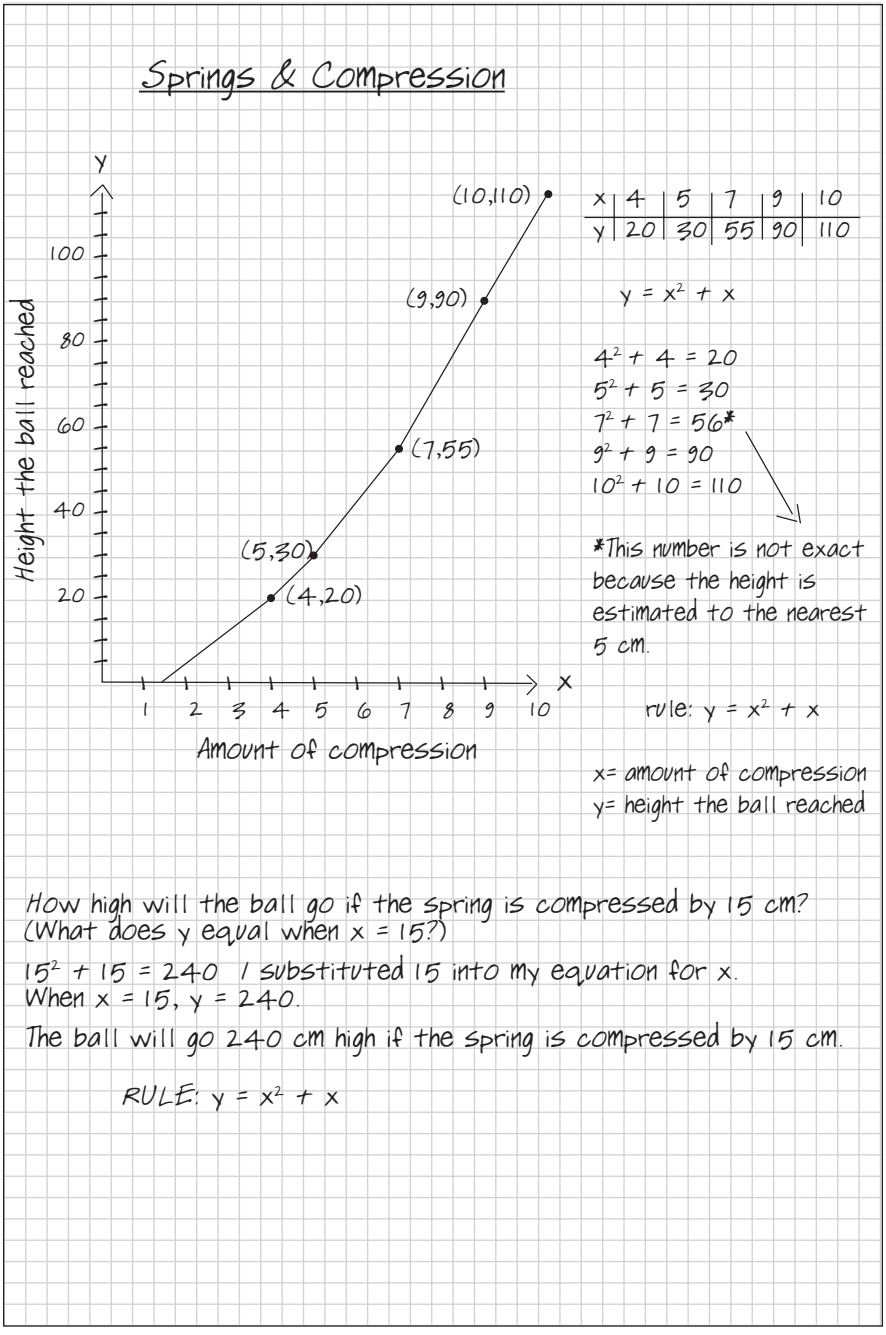
- The graph must be scaled accurately to show a continuous relationship between the compression of the spring and the height of the ball.
- The rule must be clearly described as an equation or in words and supported by the given data.
- The prediction must be clearly stated and based on the rule.

Sample Student Work for First-year Algebra

Topic: "Springs and Compression"

Score Point 4

STUDENT RESPONSE*



COMMENTARY

This response demonstrates thorough accomplishment of the task. The graph is accurately scaled, and the data points are correctly plotted as a continuous function. The rule is written as an algebraic equation and is substantiated by all data points. The prediction shows evidence of the substitution of the 15 in the student's rule.

* The student response has been typed as written, with student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

Topic: "Springs and Compression"

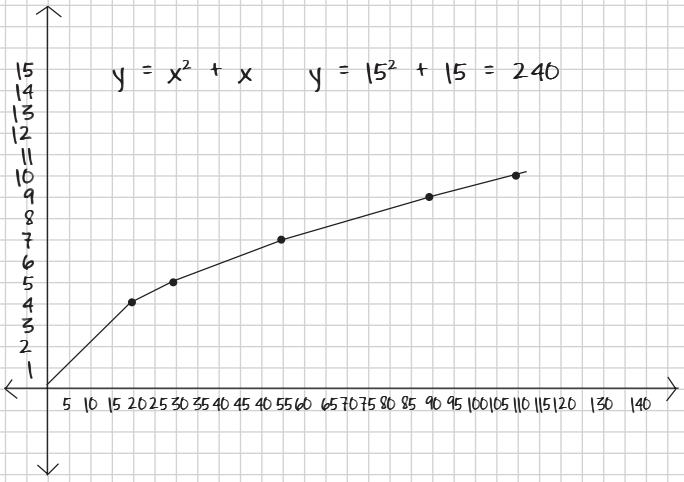
Score Point 4

STUDENT RESPONSE*

The rule, or equation, of this graph is $y = x^2 + x$. If the spring was compressed 4 cm. and shot up 20, it would be $20 = 4^2 + 4$
This rule is constant. If the $16 + 4$
spring is compressed by 15, $20 = 20$.
the answer is 240. I made
this prediction by just using the equation $y = 15^2 + 15$

$225 + 15$

$y = \boxed{240}$



COMMENTARY

This response demonstrates thorough accomplishment of the task. The graph has been accurately scaled, and the data points have been plotted as a continuous function. The equation is substantiated by a data point and explanation. The prediction shows evidence of the substitution of 15 in the student's rule.

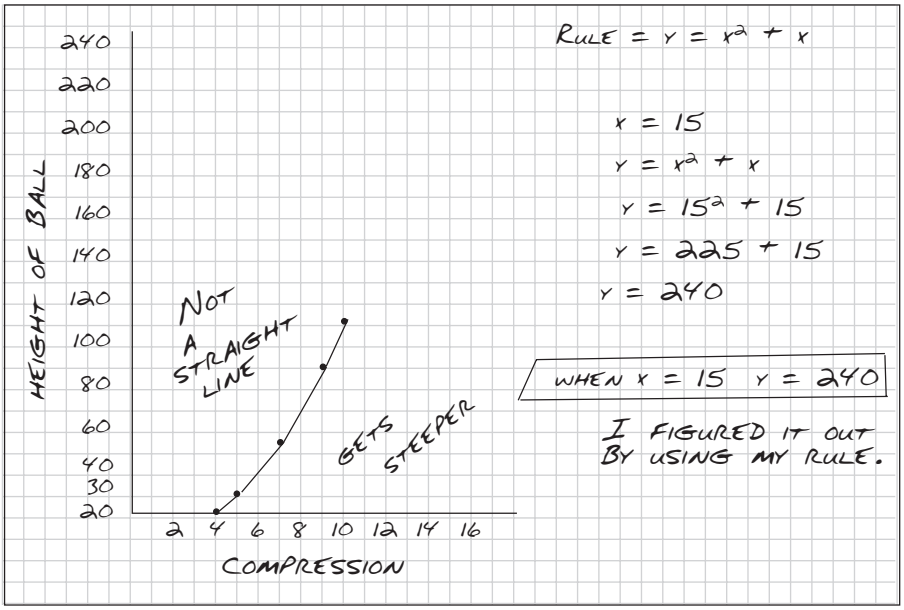
* The student response has been typed as written, with student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

Topic: "Springs and Compression"

Score Point 3

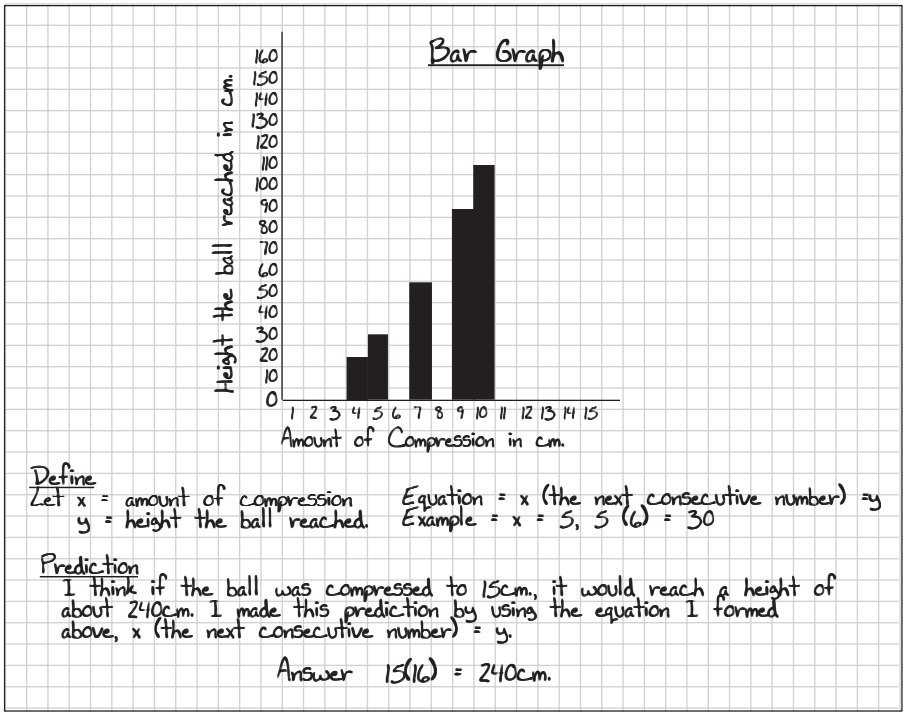
STUDENT RESPONSES*



COMMENTARY

This response substantially accomplishes the task by making a correct and accurate graph of the given data points. The rule is explained and results in a correct prediction. The student does not support the rule with any of the given data points.

Score Point 3



This response substantially accomplishes the task by using the given data to create a rule that results in an accurate prediction. There is evidence that a data point is used to substantiate the rule. A bar graph is not an appropriate representation of the given data.

* The student response has been typed as written, with student's own content, grammar, spelling, and punctuation.

Sample Multiple-choice Questions for Geometry

Directions

Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Incomplete erasures may be read as intended answers.

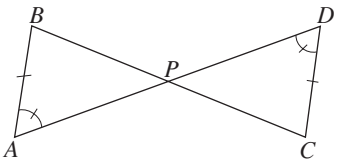
You should have available two or three No. 2 pencils with erasers and a ruler or a straightedge. You also may have a calculator, including a scientific or graphing calculator. Minicomputers, pocket organizers, or calculators with QWERTY (typewriter) keyboards are not allowed. You are not allowed to share your calculator with other students.

Work as rapidly as you can without sacrificing accuracy. Do not spend too much time on a question that seems too difficult. Answer the easier questions first; then return to the harder ones. Try to answer every question even if you have to guess.

Notes: (1) Figures that accompany problems are drawn as accurately as possible EXCEPT when it is stated that a figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(2) All numbers used are real numbers. All algebraic expressions represent real numbers unless otherwise indicated.

1. In the figure, segments $\overline{AD}$ and $\overline{BC}$ intersect at point P, $\angle A \cong \angle D$, and $AB = CD$. Which of the following can be used to prove $\Delta ABP \cong \Delta DCP$?



(Note: Figure not drawn to scale.)

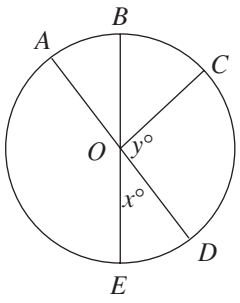
A. SSS B. SAS

C. SSA D. SAA

2. The perimeter of the triangle with vertices (0, –5), (12, 0), and (0, 5) is

A. 60 B. 48 C. 36 D. 30

3. In the figure, the circle with center O has diameters $\overline{AD}$ and $\overline{BE}$. If $x = 25$ and $y = 93$, what is the degree measure of arc BC?

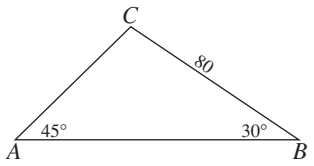


(Note: Figure not drawn to scale.)

A. 25 B. 45

C. 62 D. 65

4. In the figure $BC = 80$. What is the length of AC?



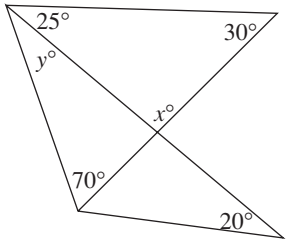
(Note: Figure not drawn to scale.)

A. $40\sqrt{2} \approx 56.57$
B. $40\sqrt{3} \approx 69.28$
C. $40\sqrt{6} \approx 97.98$
D. 40

5. In ΔABC, $m\angle A = 80°$ and $m\angle B = 65°$. Which side of the triangle is shortest?

A. $\overline{AB}$
B. $\overline{AC}$
C. $\overline{BC}$
D. The triangle has two equal sides, each of which is shorter than the third side.

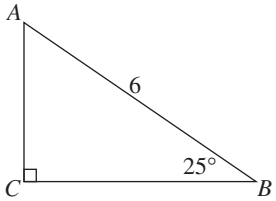
6. In the figure, $x + y =$



(Note: Figure not drawn to scale.)

A. 235
B. 180
C. 150
D. 125

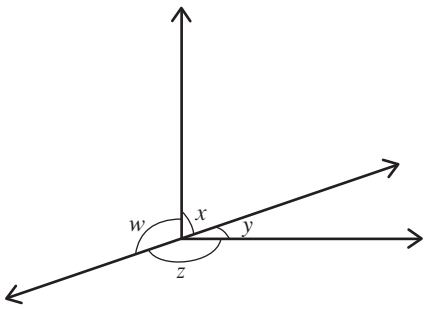
7. In the figure, $BC =$



(Note: Figure not drawn to scale.)

A. $\dfrac{6}{\cos 25°}$
B. $6 \sin 25°$
C. $6 \cos 25°$
D. $\dfrac{6}{\sin 25°}$

8. In the figure, $\angle x$ and $\angle y$ are complementary, $\angle y$ and $\angle z$ are supplementary, and the measure of $\angle y$ is 24°. What is the measure of $\angle w$?



(Note: Figure not drawn to scale.)

A. 66° B. 94°

C. 114° D. 156°

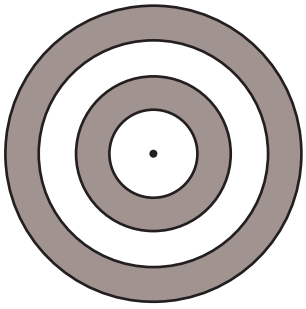
9. What polygon is created by the lines $x + 3y = 1$, $x + 3y = 4$, the x-axis and the y-axis?

A. triangle B. rectangle C. parallelogram D. trapezoid

10. If the ratio of the volumes of two cubes is 64 to 125, find the ratio of the area of a face of the small cube to the area of a face of the large cube.

A. $\frac{4}{25}$ B. $\frac{64}{125}$ C. $\frac{16}{25}$ D. $\frac{4}{5}$

11. In the figure, four circles have the same center. If the radii of the circles are 1, 2, 3, and 4, respectively, what is the total area of the shaded regions?

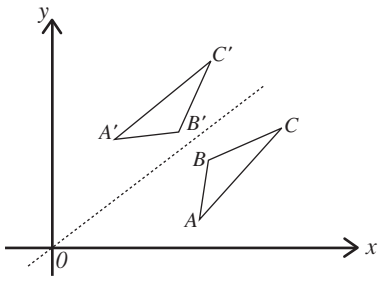


(Note: Figure not drawn to scale.)

A. $2\pi \approx 6.28$ B. $5\pi \approx 15.72$

C. $9\pi \approx 28.27$ D. $10\pi \approx 31.42$

12. In the figure, ΔABC has been reflected across the line $y = x$. If the coordinates of B are (7, 5), what are the coordinates of B'?

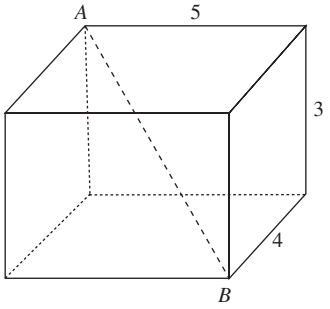


(Note: Figure not drawn to scale.)

A. (4, 7) B. (5, 2)

C. (5, 7) D. (7, 2)

13. What is the length of diagonal AB in the rectangular solid shown in the figure?



(Note: Figure not drawn to scale.)

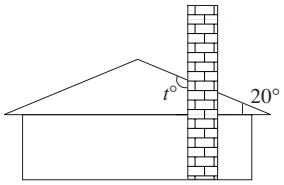
A. $\sqrt{34} \approx 5.831$ B. $5\sqrt{2} \approx 7.071$

C. $5\sqrt{3} \approx 8.660$ D. 12

14. A hole is bored through a solid metal cube with edges of length 7 inches. If the hole is a right circular cylinder with a diameter of 3 inches, what is the volume, to the nearest cubic inch, of the remaining portion of the cube?

A. 310 B. 294 C. 145 D. 49

15. The figure shows a sketch of a house for which the roof makes an angle of 20° with the horizontal and the chimney is perpendicular to the horizontal line. What is the degree measure of t?

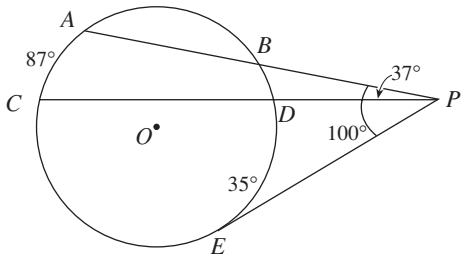


(Note: Figure not drawn to scale.)

A. 70 B. 110

C. 120 D. 160

16. Given circle O with secant $\overline{AP}$, secant $\overline{CP}$ and tangent $\overline{EP}$, $\angle APE = 100°$. The measures are as indicated in the diagram. What is the measure of arc *AB* in degrees?

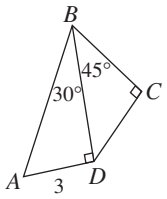


(Note: Figure not drawn to scale.)

A. 64 B. 80

C. 90 D. 134

17. Give the perimeter of quadrilateral *ABCD* to the nearest tenth.

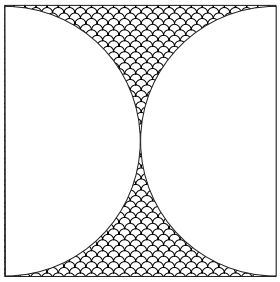


(Note: Figure not drawn to scale.)

A. 7.3 B. 16.3

C. 19.1 D. 23.1

18. The figure shown is formed by a square and two semi-circles. What is the probability that a point chosen at random from within the square falls in the shaded region?

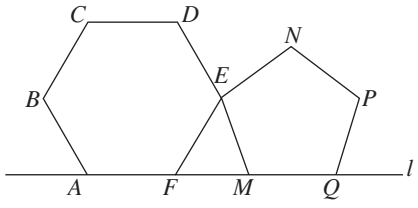


(Note: Figure not drawn to scale.)

A. 0.21 B. 0.25

C. 0.39 D. 0.50

19. In the figure, side $\overline{AF}$ of regular hexagon *ABCDEF* and side $\overline{MQ}$ of regular pentagon *MENPQ* lie on line *l*. What is the degree measure of $\angle FEM$?



(Note: Figure not drawn to scale.)

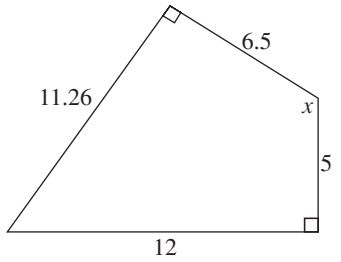
A. 114 B. 66

C. 60 D. 48

20. What is the degree measure of x?

A. 102.5 B. 120.0

C. 127.4 D. 134.8



(Note: Figure not drawn to scale.)

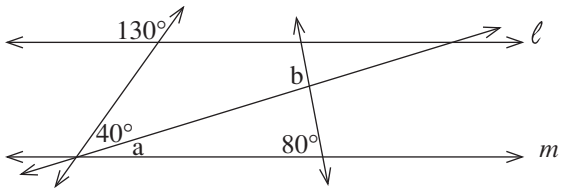
Geometry Answer Key

1. D	4. A	7. C	10. C	13. B	16. A	19. D
2. C	5. A	8. C	11. D	14. B	17. B	20. C
3. C	6. B	9. D	12. C	15. B	18. A	

Sample Written-response Problem for Geometry

Problem

In the figure below, line ℓ is parallel to line m. Explain how to find the measure of $\angle$a and the measure of $\angle$b. Specify each idea you use.



Note: Figure not drawn to scale.

Remember to show all parts of your solution, and explain how you arrived at your answer.

What Students Are Expected to Accomplish Mathematically

Students are asked to explain how to find two angles in a geometric figure, specifying each idea that they use to arrive at their answer.

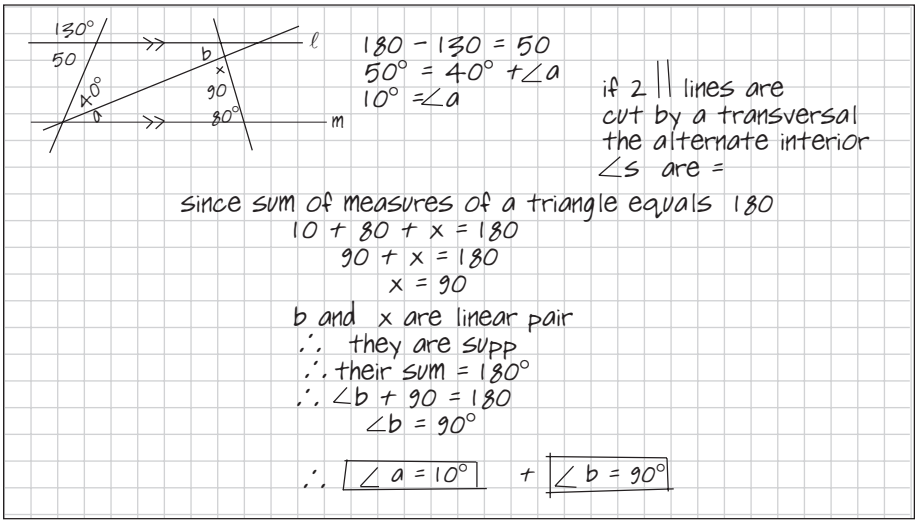
To thoroughly accomplish the task, students must clearly communicate the sequence of steps taken to reach a conclusion that the measure of $\angle a = 10°$ and the measure of $\angle b = 90°$. The response should demonstrate knowledge of angle relationships for parallel lines, intersecting lines, triangles, and/or quadrilaterals.

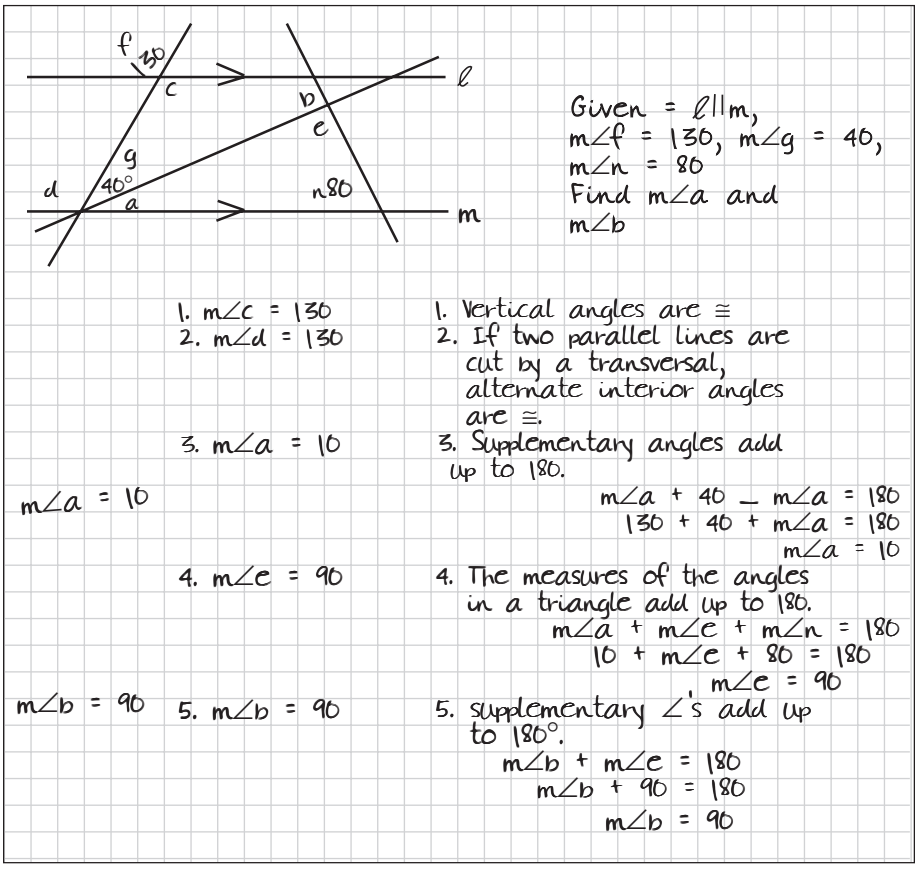
Sample Student Work for Geometry

Topic: "Angle Relationships"

Score Point 4

STUDENT RESPONSES*





COMMENTARY

This response demonstrates thorough and accurate accomplishment of the task. Both angles are calculated correctly with support given in the diagram and in the written work. Support is concise and clear with correct terminology.

This response demonstrates thorough and accurate accomplishment of the task. The work is organized and easy to follow. The use of the term "supplementary angles" in reference to three angles is a minor error in the student work. All calculations are correct and supported in a step-by-step explanation.

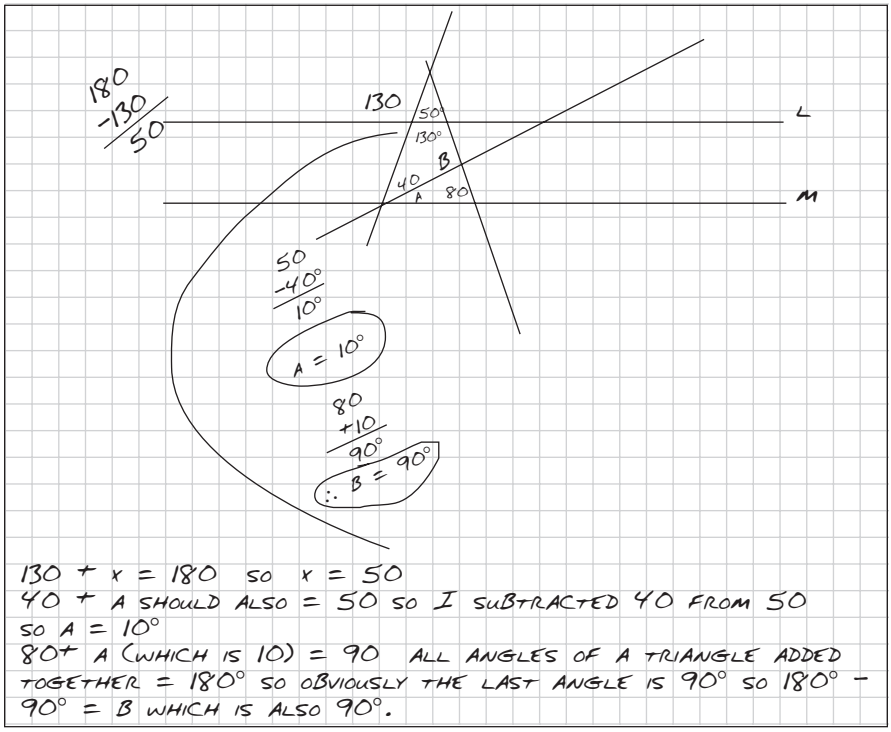
* The student response has been typed as written, with student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Topic: "Angle Relationships"

Score Point 3

STUDENT RESPONSE*



COMMENTARY

This response demonstrates substantial accomplishment of the task. Both angles are calculated correctly, but the explanation of support is not well documented (e.g., no explanation as to why "40 + A should also = 50"). The thought process is made clear.

* The student response has been typed as written, with student's own content, grammar, spelling, and punctuation.

Sample Multiple-choice Questions for High School Mathematics

1. Suppose the total daily cost C, of producing x tables, is given by $C = 3.5x + 200$. If the company produced a certain number of tables at a cost of $42,200, how many tables were produced?

   A. 147,900 B. 147,700 C. 12,114 D. 12,000

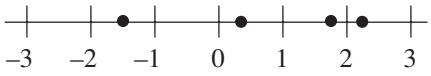
2. As treasurer for your junior class, you sold x tickets to the dance in advance and y tickets at the door. If tickets are $5 paid in advance and $6 at the door, the total amount you collected can be represented as

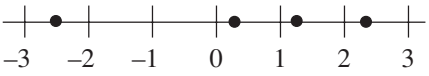
   A. $30xy$ B. $11(x + y)$ C. $30(x + y)$ D. $5x + 6y$

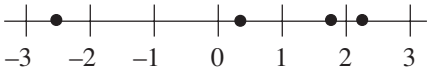
3. Kevin's grandmother placed $1,000 in a savings account on the day he was born in 1982. It has been drawing a 9% interest rate compounded annually. How much should Kevin have on his birthday in the year 2000 if the interest remained constant every year?

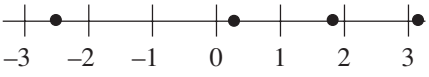
   A. $4,327.63 B. $4,717.12 C. $19,620.00 D. $34,200.00

4. Which of the following number lines represents the graph of $\frac{1}{3}, -\frac{5}{2}, \sqrt{5}, 1.8$?

   A.
   

   B.
   

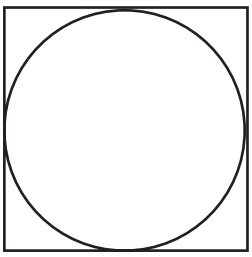
   C.
   

   D.
   

5. A computer company's change in the number of employees for three consecutive years is, respectively, 20% increase, 30% increase, and 20% decrease. What is the total percent change (rounded to the nearest percent)?

   A. 25% B. 30% C. 70% D. 87%

6.



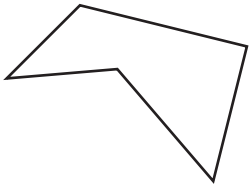
If the area of the given square is 25, what is the area of the inscribed circle?

A. 5π B. $\frac{25\pi}{4}$ C. 10π D. 25π

7. The function $f(x)$ is an increasing function and has a single asymptote at $y = 5$. Which of the following could be an equation for $f(x)$?

A. $f(x) = 2x + 6$ B. $f(x) = \left(\frac{1}{2}\right)^x + 5$ C. $f(x) = 2^x + 5$ D. $f(x) = 2^x + 6$

8.



The figure shown above has an area of six square units. A similar figure is constructed so that each segment of the boundary is three times as long as the corresponding side in the figure shown. The area of the new figure, in square units, is

A. 18 B. 36 C. 54 D. 108

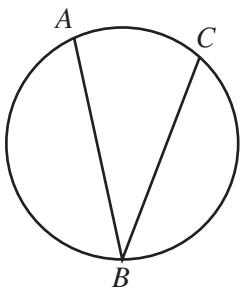
9. In the first-period algebra class, the average test score was 73.4, and in the second-period class, the average score was 69.4. There were 31 students in the first-period class and 37 in the second-period class. What was the average score for the two classes combined?

A. 71.2 B. 71.3 C. 71.4 D. 71.5

10. A bag contains four one-dollar bills, two five-dollar bills, and two ten-dollar bills. Two randomly selected bills are removed from the bag. What is the probability that neither bill is a ten-dollar bill?

A. $\frac{2}{7}$ B. $\frac{15}{28}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$

11.



The figure shown above is a circle with a radius of six units, and the measure of ∠ABC is 30°. The length of arc AC is approximately

A. 3 units B. 6 units C. 19 units D. 108 units

12. Which of the following equations of a parabola has a vertex at (–4, 16) and passes through the origin?

A. $y = (x - 4)^2 + 16$ B. $y = (x + 4)^2 + 16$ C. $y = -(x - 4)^2 + 16$ D. $y = -(x + 4)^2 + 16$

High School Mathematics Answer Key

1. D	4. C	7. C	10. B
2. D	5. A	8. C	11. B
3. B	6. B	9. A	12. D

Sample Written-response Problem for High School Mathematics

Problem

Teresa needs your advice. She is considering a one-time, long-term investment with no withdrawals. Her banker presents her with two options:

Option 1	Option 2
Invest $10,500 at 3% interest compounded annually	Invest $10,000 at 4% interest compounded annually

She wishes to choose the option that will give her the most money. Advise Teresa on which option to take.

A complete response will include your answer to the question, all parts of your solution shown, and an explanation of how you arrived at your answer.

What Students Are Expected to Accomplish Mathematically

To thoroughly accomplish the task in this problem, students must describe how the amount of money for each option is increasing and identify the point (year) at which one option is more favorable than the other. A table or other grid may be used to display the data. Students should include the mathematical computations used to develop the table or grid.

Sample Student Work for High School Mathematics

Topic: "Teresa Needs Advice"

Score Point 4

STUDENT RESPONSE*

Year	Option 1 ($)	Option 2 ($)
0	10,500	10,000
1	(10,500 • 1.03:) 10.815	(10,000 • 1.04) 10,400
2	(10,815 • 1.03) 11139.45	(10,400 • 1.04) 10,816
3	11473.63	11248.64
4	11817.84	11698.54
5	12172.38	12166.53
6	12537.55	12653.19

Option 1 makes more in the beginning, however option 2 slowly catches up until it makes more in Year 6.

Assuming Long Term means more than 6 years, Option 2 is the better deal.

COMMENTARY

This response shows a thorough understanding of how the amount of money for each option is increasing. The table clearly shows how the numbers were computed, although a function is not identified. The response correctly identifies the point at which option #2 becomes more favorable and gives a definition of *long-term* necessary to answer the question. Mathematical computations are appropriate and accurate.

* The student response has been typed as written, with student's own content, grammar, spelling, and punctuation.

Sample Student Work for High School Mathematics

Topic: "Teresa Needs Advice"

Score Point 3

STUDENT RESPONSE*

OPTION 1

10 yrs. ① 10,500 (3/100) =) 10,815 ⑥ 12172.38 (3/100) =) 12537.55
② 10815 (3/100) =) 11139.45 ⑦ 12537.55 (3/100) =) 12913.13
③ 11139.45 (3/100)=) 11473.63 ⑧ 12913.13 (3/100) =) 13300.52
④ 11473.63 (3/100) =) 11817.84 ⑨ 13300.52 (3/100) =) 13699.54
⑤ 11817.84 (3/100) =) 12172.38 ⑩ 13699.54 (3/100) =) 14110.51

OPTION 2

10 yrs. ① 10000 (4/100) =) 10,400 ⑥ 12166.53 (4/100) =) 12653.22
② 10400 (4/100) =) 10,816 ⑦ 12653.22 (4/100) =) 13159.35
③ 10816 (4/100)=) 11248.64 ⑧ 13159.35 (4/100) =) 13685.72
④ 11248.64 (4/100) =) 11698.59 ⑨ 13685.78 (4/100) =) 14233.15
⑤ 11698.59 (4/100) =) 12166.53 ⑩ 14233.15 (4/100) =) 14802.48

By doing this problem by calculator I found out that Teresa could make more by investing in option 2 in 10yrs. she could have 691.97 dollars more than the option 1 plan

COMMENTARY

This response uses a table of values to determine years 1 to 10, showing an essential grasp of the mathematical idea. The response in large part clearly chooses the correct options but fails to point out the importance of intersection points. Mathematical computations are appropriate and generally accurate.

* The student response has been typed as written, with student's own content, grammar, spelling, and punctuation.